EE-434 Power Electronics

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Overview

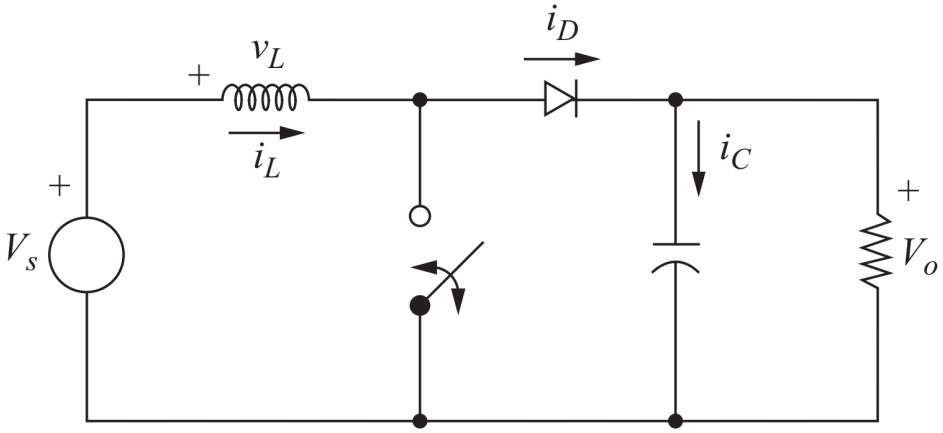
1. DC-DC Boost converter
   - Assumptions for analysis
   - Analysis for switch closed
   - Analysis for switch open
   - Transfer function derivation
   - Determining the component values
     - Inductance value
     - Capacitor value
   - What happens when the converter first starts?
   - Design Examples
     - Simulation results of Example 6-4
2. Comparison of buck and boost converter

Introduction

The output voltage is always greater than the input voltage and hence it is called a boost converter.

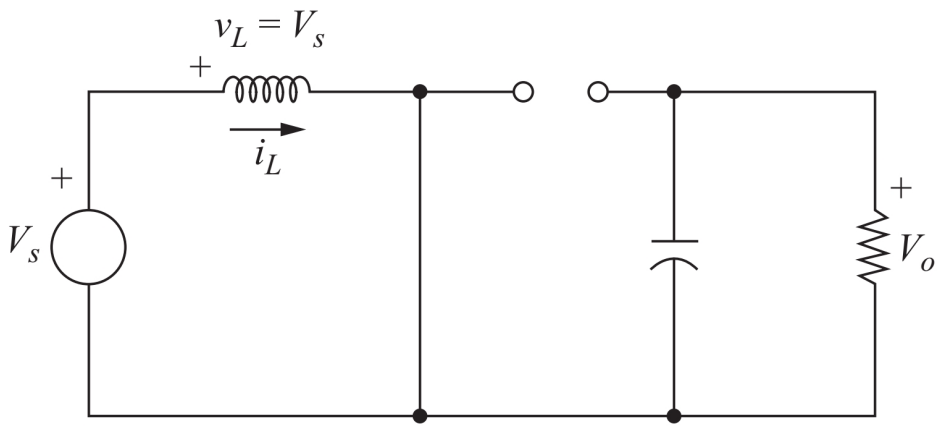
Analysis is based on following.

- Steady-state conditions exist.
- Switching period is T, switch is closed for DT and open for (1-D)T
- Inductor is in CCM
- Output capacitor is large enough so that the voltage (V_o) is essentially constant.
- All components are ideal.



Analysis for switch closed $0 \leq T \leq DT$

The equivalent circuit with switch closed is shown below. Diode is reverse biased.



Applying KVL around the closed path

$$v_L = V_s = L\frac{di_L}{dt} \implies \frac{di_L}{dt} = \frac{V_s}{L} \tag{1}$$

Current increases linearly during this duration. The current starts from an initial value and ends at some value. So that the change in inductor current is given as

$$(\Delta i_L)_{closed} = \frac{V_s DT}{L} \tag{2}$$

Analysis for switch closed $0 \leq T \leq DT$

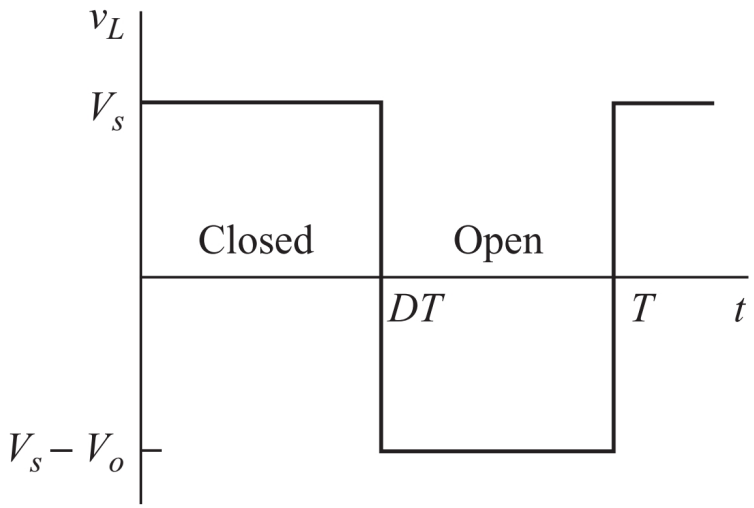


Figure: Inductor voltage

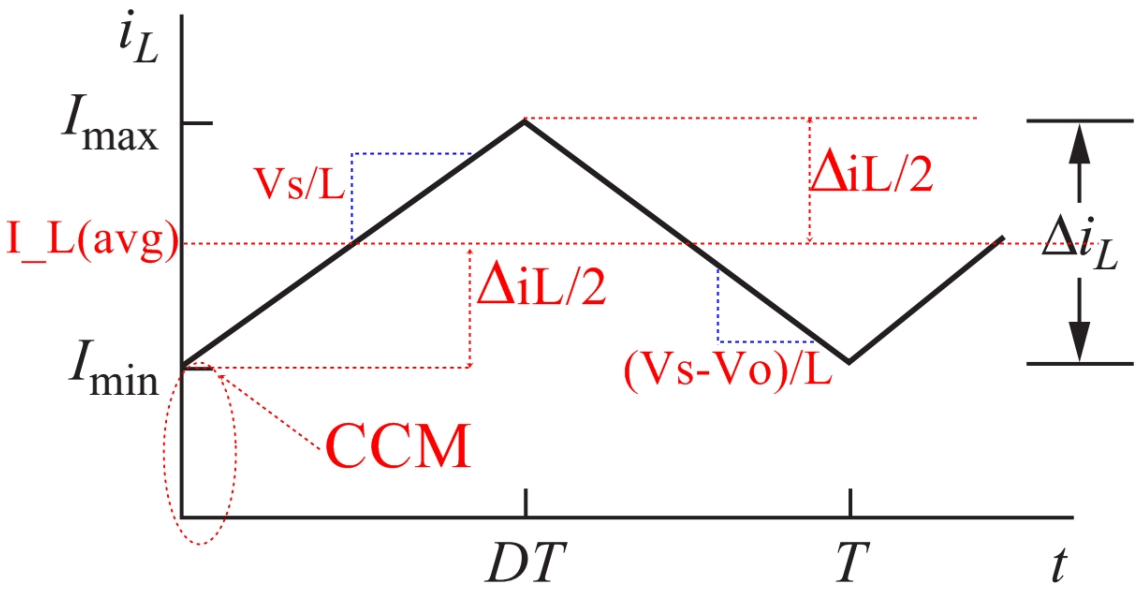
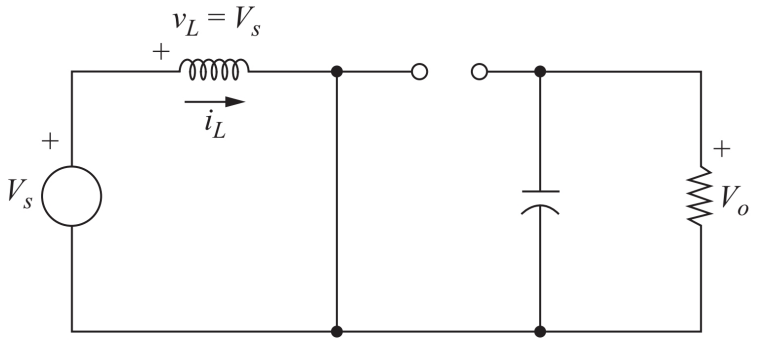


Figure: Inductor current

Analysis for switch open $DT \leq T \leq T$

The equivalent circuit with switch open is shown below. Diode is forward biased because inductor current needs a path.



Applying KVL around the closed path

$$v_L = V_s - V_o = L\frac{di_L}{dt} \implies \frac{di_L}{dt} = \frac{V_s - V_o}{L} \tag{3}$$

Current decreases linearly during this duration. The current starts from the peak value and ends at some value. So that the change in inductor current is given as

$$(\Delta i_L)_{open} = \frac{(V_s - V_o)(1 - D)T}{L} \tag{4}$$

Analysis for switch closed $DT \leq T \leq T$

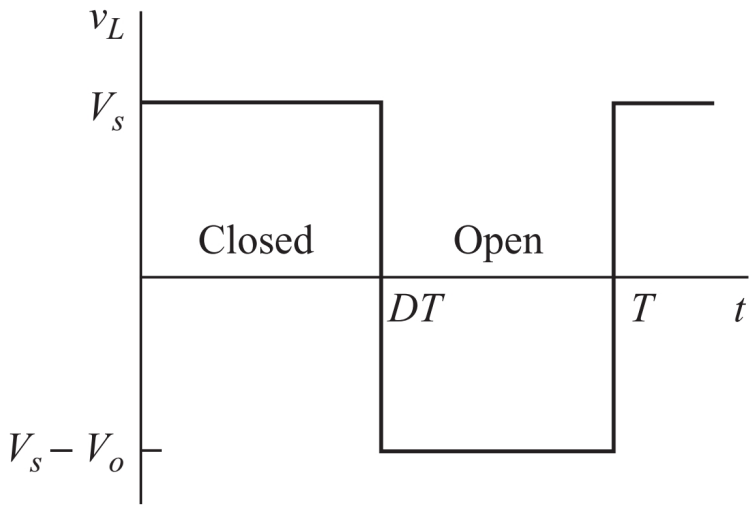


Figure: Inductor voltage

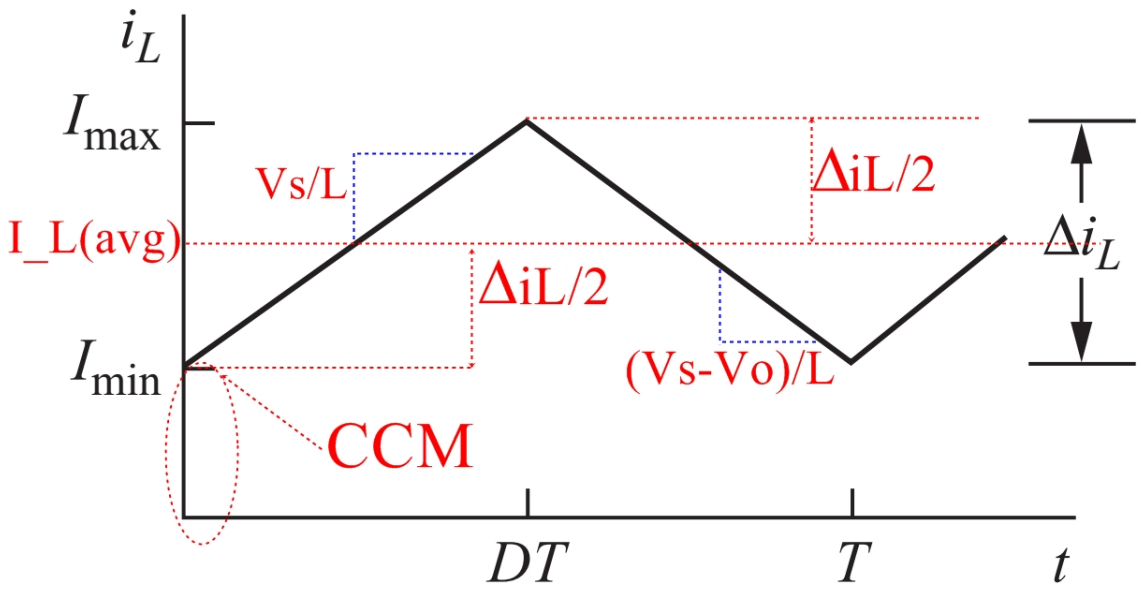


Figure: Inductor current

Transfer function derivation

For steady state operation, the change in inductor current should be zero. Using (2) and (4)

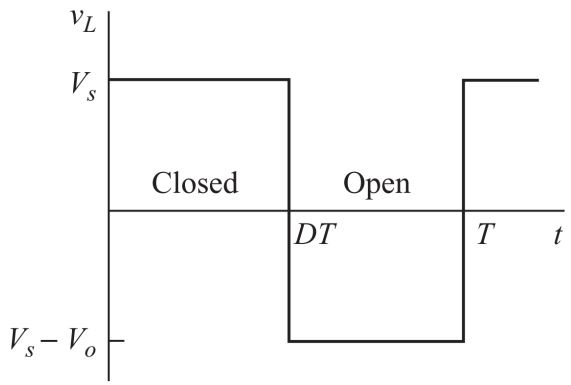
$$\underbrace{(\Delta i_L)_{closed}}_{\frac{V_s D T}{L}} + \underbrace{(\Delta i_L)_{open}}_{\frac{(V_s - V_o)(1-D)T}{L}} = 0 \tag{5}$$

Solving for V_o,

$$V_s(D + 1 - D) - V_o(1 - D) = 0 \implies V_o = \frac{V_s}{1 - D} \tag{6}$$

Transfer function derivation

Transfer function can also be computed by equating the volt-sec product equal to zero.



$$V_L = V_s D + (V_s - V_o)(1 - D) = 0 \tag{7}$$

Solving for V_o,

$$V_o = \frac{V_s}{1 - D} \tag{8}$$

Observations

Output voltage is not a linear function. For D=1 the output tends to infinity however, non-ideality of components does not let this happen.

Transfer function derivation

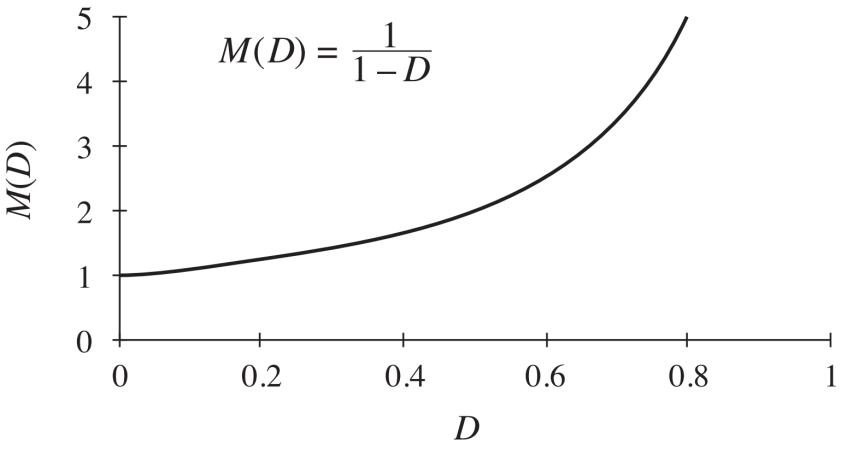


Figure: Transfer characteristics of ideal boost converter

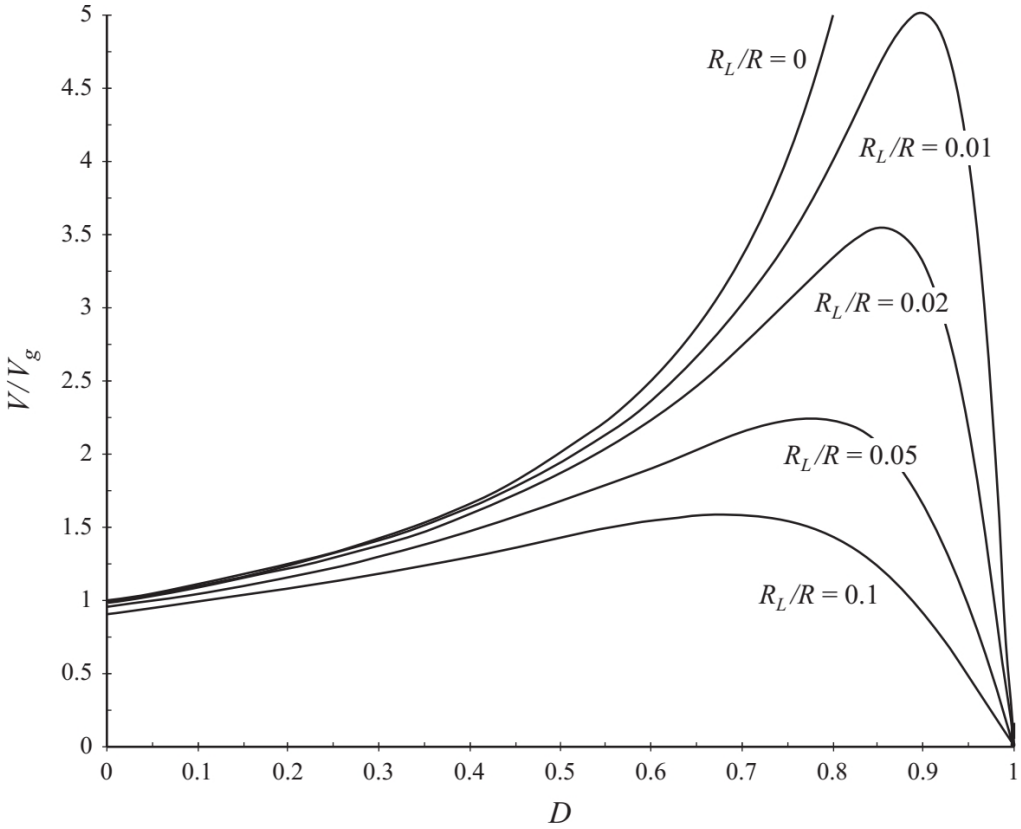


Figure: Transfer characteristics of boost converter with finite resistance in inductor

Determining the component values

For a lossless converter

$$\underbrace{P_{in}}_{V_s I_s = V_s I_L} = \underbrace{P_o}_{V_o I_o} \tag{9}$$

Equating and substituting (8)

$$V_s I_L = \frac{V_o^2}{R} = \frac{(V_s/(1-D))^2}{R} = \frac{V_s^2}{(1-D)^2} R \tag{10}$$

Solving for average inductor current

$$I_L = \frac{V_s R}{(1-D)^2 R} = \frac{V_o^2}{V_s R} = \frac{V_o I_o}{V_s} \tag{11}$$

Maximum and minimum inductor currents are determined by

$$I_{max} = I_L + \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} + \frac{V_s D T}{2L} \tag{12}$$

$$I_{min} = I_L - \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} - \frac{V_s D T}{2L} \tag{13}$$

Determining the component values

To ensure CCM,

$$I_{min} = \frac{V_s}{(1-D)^2 R} - \frac{V_s DT}{2L} = 0 \tag{14}$$

or

$$\frac{V_s}{(1-D)^2 R} = \frac{V_s DT}{2L} = \frac{V_s D}{2Lf} \tag{15}$$

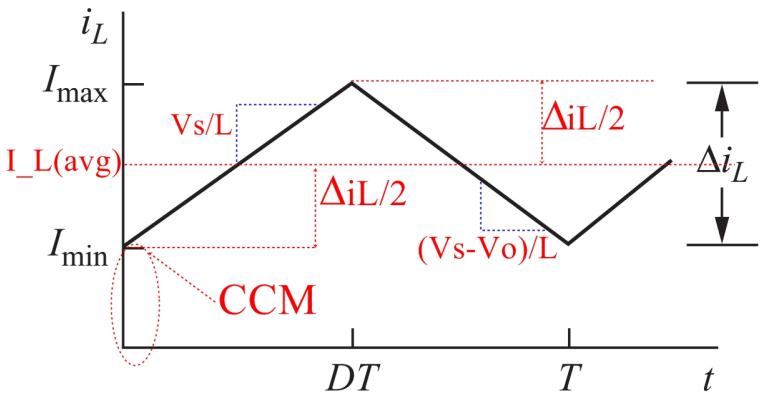
Minimum combination of inductance and switching frequency in CCM is

$$(Lf)_{min} = \frac{D(1-D)^2 R}{2} \tag{16}$$

$$L_{min} = \frac{D(1-D)^2 R}{2f} \tag{17}$$

Determining the component values

For a design perspective inductance is usually expressed in terms of the desired current ripple. The method no.2 presented in Week 9 is used. Consider the following



For the On time,

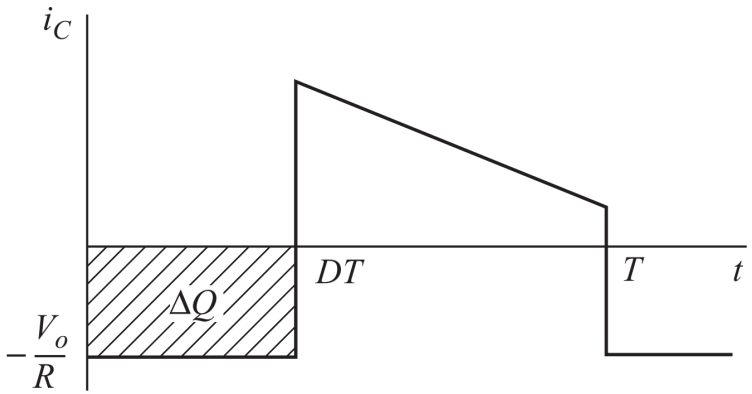
$$\Delta I_L = slope \times DT = \frac{V_s}{L} DT = 0 \tag{18}$$

Rearranging,

$$L = \frac{V_s DT}{\Delta i_L} = \frac{V_s D}{\Delta i_L f} \tag{19}$$

Determining the component values

Peak to peak output ripple can be calculated using capacitor waveform



$$|\Delta Q| = \left(\frac{V_o}{R}\right) DT = C\Delta V_o \quad (20)$$

The ripple voltage is

$$\Delta V_o = \frac{V_o DT}{RC} = \frac{V_o D}{RCf} \quad (21)$$

or

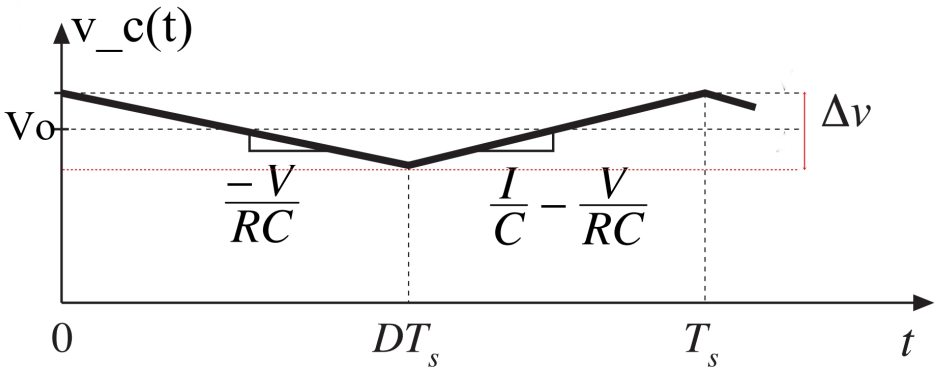
$$\frac{\Delta V_o}{V_o} = \frac{D}{RCf} \quad (22)$$

Capacitor value is given as

$$C = \frac{D}{R(\Delta V_o / V_o) f} \quad (23)$$

Determining the component values

The capacitor voltage swings around an average value as shown below



Using the current through a capacitor the slope of the capacitor voltage can be expressed as $\frac{dv}{dt} = \frac{i_c}{C}$

$$\frac{dv_c(t)}{dt} = \frac{i_c(t)}{C} = \frac{-V}{RC} \tag{24}$$

Change in capacitor voltage is given as

$$slope \times DT \implies -2\Delta V_o = \frac{-V_o}{RC} DT \tag{25}$$

$$C = \frac{V_o D}{fR\Delta v_o} \tag{26}$$

What happens when the converter first starts?

The inductor current is a straight line for the first few switching cycles because the output voltage is negligible.

Student exercise

SImulation is FUN. try it at home

Example 6-4

Scenario

Design a boost converter that will have an output of 30 V from a 12-V source. Design for continuous inductor current and an output ripple voltage of less than one percent. The load is a resistance of 50Ω. Assume ideal components for this design.

Solution:
Using (8) D=1-$\frac{V_s}{V_o}$=0.6
Selecting a switching frequency f=25kHz.
The minimum inductance is given in (17)

$$L_{min} = \frac{D(1-D)^2 R}{2f} = 96\mu H \tag{27}$$

To ensure CCM, let L=120μH.
Using (11) and (2)

$$I_L = \frac{(V_s}{(1-D)^2 R} = 1.5A \tag{28}$$

Example 6-4

$$\frac{\Delta i_L}{2} = \frac{V_s DT}{2L} = 1.2A \tag{29}$$

This means

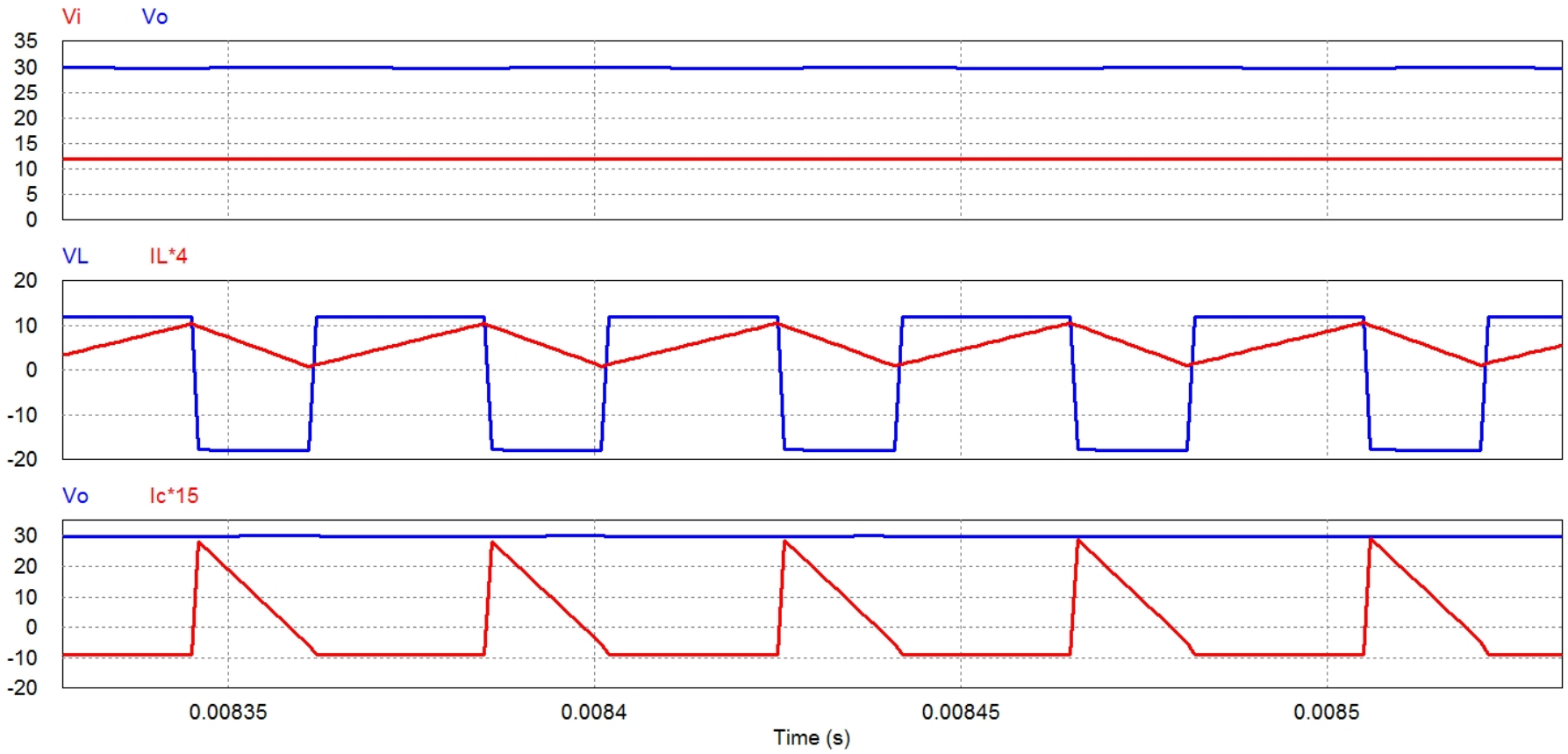
$$I_{max} = 1.5 + 1.2 = 1.7A \tag{30}$$

$$I_{max} = 1.5 - 1.2 = 0.3A \tag{31}$$

Capacitor value is given as

$$C \geq \frac{D}{R(\Delta V_o / V_o) f} = 48\mu F \tag{32}$$

Simulation results of Example 6-4



Vi and Vo are input and output voltage respectively. VL is the inductor voltage, IL is inductor current (multiplied by 4 for easy visualization) Ic is capacitor current(multiplied by 15 for easy visualization)

Example 6-5

Please go through this example yourself and remember SIMULATION is FUN :)

Comparison of buck and boost converter

Specifications	Buck	Boost
Switch parameters	01 $V_{DSmin} = V_{in}$	01 $V_{DSmin} = V_{in}$
Diode	01 $V_{RRmin} = V_{in}$	01 $V_{RRmin} = V_{in}$
Capacitor	01 $V_C \geq V_o$	01 $V_C \geq V_o$
Inductor	01 $I_L \geq I_{rms}$	01 $I_L \geq I_{rms}$
Gate driver	High side	Low side
Current[1]	Discontinuous input ,continuous output	Continuous input ,discontinuous output
cell7	cell8	cell9

[1] Important in PV system design